Divide-and-Conquer Determinization of Büchi automata

Yong Li

Joint work with Andrea Turrini, Weizhi Feng, Moshe Y. Vardi and Lijun Zhang
Büchi determinization

Büchi automata are not closed under determinization

Nondeterministic Büchi automata (NBA) → Deterministic ω-automata
- Rabin (DRA)
- Parity (DPA)
- Emerson-Lei (DELA)
Why Büchi determinization is important

➢ Reactive synthesis
➢ Probabilistic verification
➢ Complementing NBA
➢ Checking language inclusion of NBAs
Existing constructions

Input NBA

- SCC 1
- SCC 2
- ... 
- SCC k

Deterministic $\omega$-automata
- Rabin (DRA)
- Parity (DPA)

Safra-Piterman’s construction
Existing constructions

Input NBA

SCC 1

SCC 2

... 

SCC k

Deterministic $\omega$-automata

- Rabin (DRA)
- Parity (DPA)

Work on all SCCs at once
Our contributions

1. Divide-and-conquer methodology

2. Two subclasses with better upper bounds

3. Comprehensive evaluation
Our determinization construction

Input NBA $A$

SCC decomposition

$\text{SCC 1}$ $\longrightarrow$ $\text{DPA 1}$
$\text{SCC 2}$ $\longrightarrow$ $\text{DPA 2}$
$\vdots$
$\text{SCC K-1}$ $\longrightarrow$ $\text{DPA K-1}$
$\text{SCC k}$ $\longrightarrow$ $\text{DPA k}$

Union product

DELÁ
Insight 1:
Determine each SCC independently
Determinizing different types of SCCs

Three different types of SCCs

1. Inherently Weak SCC (IWC): $3^n$

2. Deterministic Accepting SCC (DAC): $O(n!)$

3. Nondeterministic Accepting SCC (NAC): $O((n!)^2)$
Determinizing different types of SCCs

Three different types of SCCs

Insight 2:
Specific construction for each type of SCCs
Final determinization construction

Input NBA $A$

SCC decomposition

Perform union product \textit{on-the-fly}

DPA $n_d$

DPA $n_c$

DPA $n_w$

Union product

DELA
Empirical evaluation

**COLA** solves **more instances in shorter time**

- **Number of solved cases**
  - COLA: 17,351
  - Spot: 67,258
  - Owl: 206,431

**PAR-2 score:** lower is better

- COLA: 17,351
- Spot: 67,258
- Owl: 206,431
Comparison with Spot

Heat map: blue color corresponds to fewer data points

COLA constructs smaller deterministic automata than Spot
Comparison with Owl

Heat map: blue color corresponds to fewer data points

COLA constructs smaller deterministic automata than Owl
Summary

1. **Divide-and-conquer** determinization
2. Better upper bounds for two subclasses:
   - $O(n!)$ vs. $O((n!)^2)$ and $O(2^n)$ vs. $O(n!)$
3. COLA outperforms **Spot** and **Owl**

Future work
- **Parallel** determinization for each SCC
- Applications to
  - Reactive synthesis
  - Probabilistic verification
  - Büchi complementation and inclusion